TopMath.Info
Math Glossary

Version 0.86

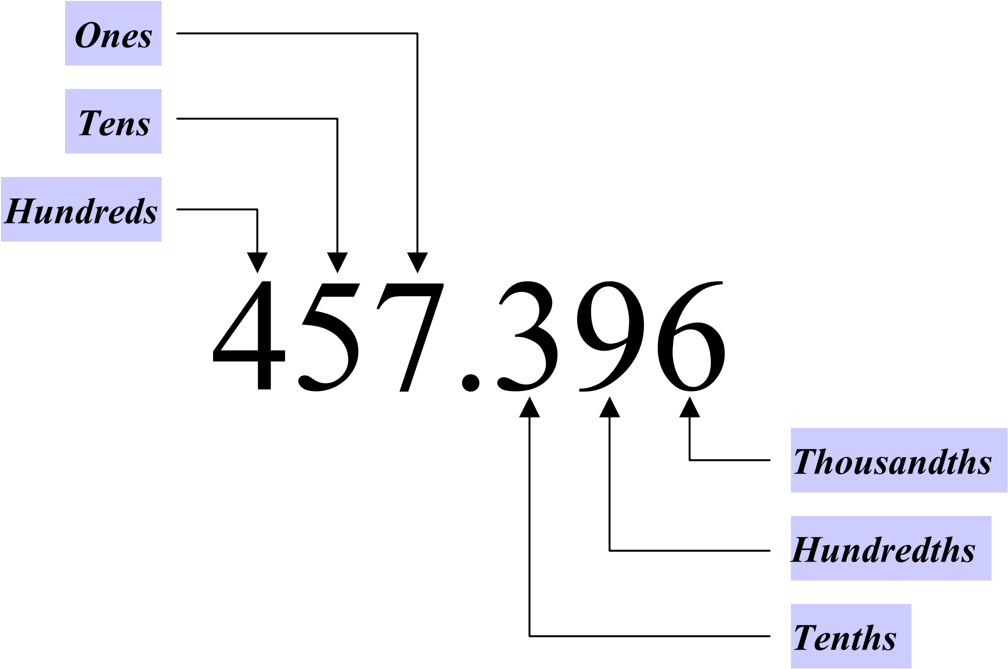
August 27, 2006

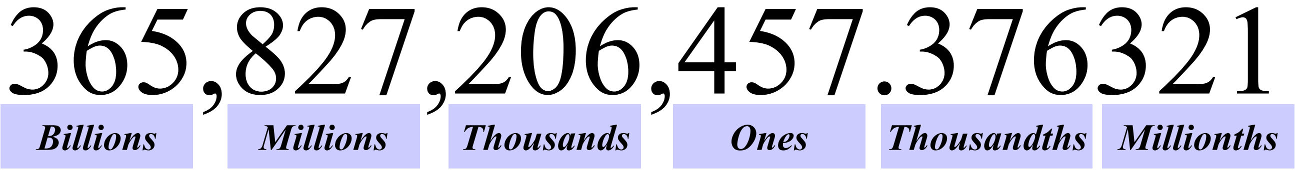
Table of Contents

Types of Numbers

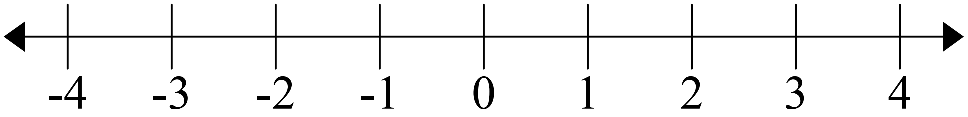
Whole Numbers	1, 2, 3, 4, . . .	Positive, no decimal points
Integers	. . . -3, -2, -1, 0, 1, 2, 3, . . .	Positive and negative whole numbers and 0.
Even Numbers	. . . -6, -4, -2, 0, 2, 4, 6, . . .	End in 0, 2, 4, 6, or 8
Odd Numbers	. . . -5, -3, -1, 1, 3, 5, . . .	Integers ending in 1, 3, 5, 7, or 9
Rational Numbers	1, 0.5, 2/3, $0.\overline{123}$	Can be written as a fraction of two integers Either stop or have repeating digits to right of decimal point
Irrational Numbers	π, $\sqrt{2}$, 0.121121112 . . .	Cannot be written as a fraction of two integers Go on forever to right of decimal point without repeating

Place Value


Ones
Tens
Hundreds
457.396
Thousandths
Hundredths
Tenths


365,827,206,457.376321
Billions
Millions
Thousands
Ones
Thousandths
Millionths

The Number Line



Negative Numbers
(less than 0)

Positive Numbers
(greater than 0)

$3 + -2 = 1$

$3 - -2 = 5$

Adding a negative number is like subtracting a positive number.

Subtracting a negative number is like adding a positive number.

$3 \times -4 = -12$

Multiplying a positive number by a negative number produces a negative number.

$-3 \times -4 = 12$

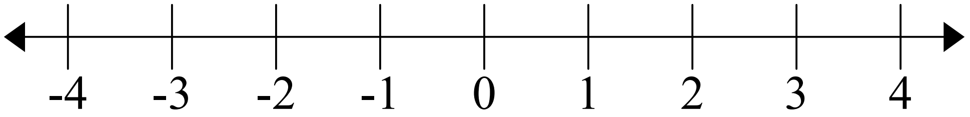
Multiplying two negative numbers produces a positive number.

The ***absolute value*** of a number is its distance from zero, regardless of its sign:

The symbol for absolute value is a pair of vertical lines around the number:

$| -3 | = 3$
$| 20 | = 20$
$| 0 | = 0$

Equations and Inequalities



Negative Numbers
(less than 0)

Positive Numbers
(greater than 0)

$3 = 2+1$

An ***equation*** uses an *equal sign* (=) to mean ***equal to***

$2 < 4$

$-4 < -2$

A number is always less than any number to the right of it on the number line.

$7 > 3+3$

$5 < 4+3$

An ***inequality*** often uses *greater than* (>) or *less than* (<) to indicate that which of two quantities is greater.

There are three other symbols used in inequalities:

$\geq$ greater than or equal to
$\leq$ less than or equal to
$\neq$ not equal to

Arithmetic

Addition

Carry

$$\begin{array}{r} {}^{1} \\ 23 \\ +59 \\ \hline 82 \end{array}$$

Addends

Sum

Subtraction

Borrow

$$\begin{array}{r} {}^{8}\cancel{9}{}^{1}4 \\ -26 \\ \hline 68 \end{array}$$

Difference

In the examples above, it appears that the number being carried or borrowed is a 1. In fact, it is a 10, because it is a 1 in the tens column.

Multiplication

$$\begin{array}{r} 23 \\ \times 12 \\ \hline 46 \\ 23 \\ \hline 276 \end{array}$$

ctors

Product

Division

Quotient

Remainder

$$\begin{array}{r} 14R5 \\ 8\overline{)\,117} \end{array}$$

Divisor

Dividend

There are four ways to show multiplication:

3 x A

3 • A

3A

3 (A)

There are three ways to write the quotient shown above:

14 R 5

$14\ {}^{5}/_{8}$

14.625

You cannot divide any number by zero.

Divisibility

One whole number is ***divisible*** by another if the second divides into the first evenly (with a remainder of 0).

Number	Rule	Examples
2	A number is divisible by 2 if and only if **it ends in 0, 2, 4, 6, or 8.**	•34 is divisible by 2 because it ends in 4. •43 is not divisible by 2 because it ends in 3.
3	A number is divisible by 3 if and only if **the sum of its digits is divisible by 3.**	•264 is divisible by 3 because 2+6+4 = 12, which is divisible by 3. •325 is not divisible by 3 because 3+2+5 = 10, which is not divisible by 3.
5	A number is divisible by 5 if and only if **it ends in 0 or 5.**	•65 is divisible by 5 because it ends in 5. •501 is not divisible by 5 because it ends in 1.
6	A number is divisible by 6 if and only if **it is divisible by 2 and 3.**	•354 is divisible by 6 because it ends in 4 and 3+5+4 = 12, which is divisible by 3. •562 is not divisible by 6 because 5+6+2 = 13, which is not divisible by 3.
9	A number is divisible by 9 if and only if **the sum of its digits is divisible by 9.**	•387 is divisible by 9 because 3+8+7 = 18, which is divisible by 9. •496 is not divisible by 9 because 4+9+6=19, which is not divisible by 9.
10	A number is divisible by 10 if and only if **it ends in 0.**	•370 is divisible by 10 because it ends in 0. •7003 is not divisible by 10 because it does not end in 0.

A whole number is ***prime*** if it is greater than 1 and it is only divisible by 1 and itself. The first few prime numbers are 2, 3, 5, 7, 11, 13, 17, and 19.

A whole number is ***composite*** if it is greater than 1 and not prime. The first few composite numbers are 4, 6, 8, 9, 10, 12, 14, 15, and 16.

Decimal Numbers

Addition

Line up decimal points

$$\begin{array}{r} 241.38 \\ +\ \ 69.17 \\ \hline 310.55 \end{array}$$

Subtraction

Line up decimal points

$$\begin{array}{r} 241.38 \\ -\ \ 69.17 \\ \hline 172.21 \end{array}$$

Multiplication

$$\begin{array}{r} 2.34 \\ \times 1.2 \\ \hline 468 \\ 234 \\ \hline 2.808 \end{array}$$

The number of digits to the right of the decimal point in the answer (product) is the sum of the number of digits to the right of the decimal point in the two factors. In this case, 2+1 = 3.

Division

$$6.2\overline{)\,121.74}$$

$$\Downarrow$$

$$62\overline{)\,1217.4}$$

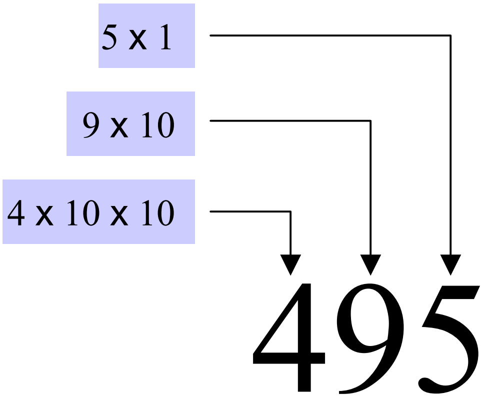
If you have a decimal point in the divisor, move it to the right until the divisor becomes an integer. Then move the decimal point in the dividend to the right the same number of places. In this case, we moved each decimal point one place to the right.

Ways of Showing Numbers

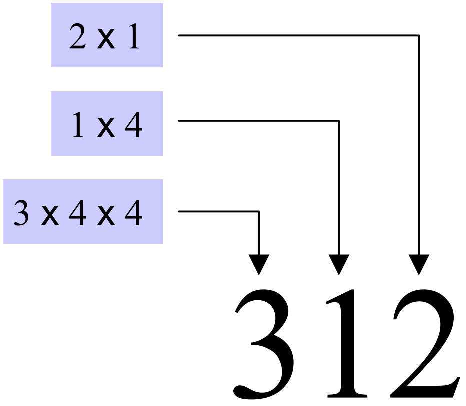
Expanded Notation	40,125 = 40,000+100+20+5
Always has one number for every digit other than 0.	
Repeating Decimals	3/11 = 0.27272727... = $0.\overline{27}$
The line goes over the repeating part.	
Exponential Notation	15,700,000 = 15.7×10^6
Usually written so that exponent is a multiple of three, indicating thousands, millions, billions, trillions, and so on.	
Scientific Notation	15,700,000 = 1.57×10^7
Similar to exponential notation, but beginning number must always be greater than or equal to 1 and less than 10.	
Roman Numerals	1 = I 5 = V 10 = X 50 = L 100 = C 500 = D 1000 = M
An awkward system of notation, of limited use today. Still helpful in understanding years engraved on buildings, and numbers of events, such as Super Bowl XXXIV. Helps people appreciate the importance of our current system of Arabic numerals.	

Number Bases

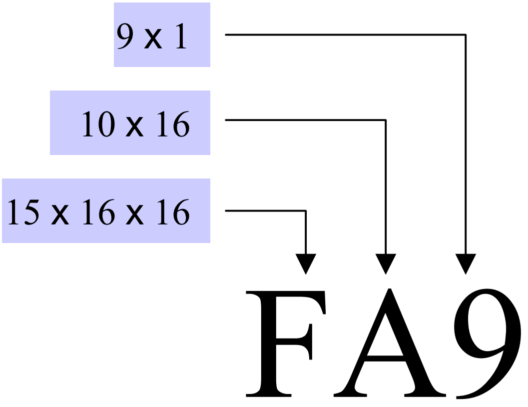
Numbers are usually written in ***base 10***, which represents numbers using combinations of the ten symbols 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9, as shown. below.



Suppose you had only four symbols, 0, 1, 2, and 3. This is called ***base 4***, and the numbers are written as shown below. This number is equivalent to 54 in base 10.

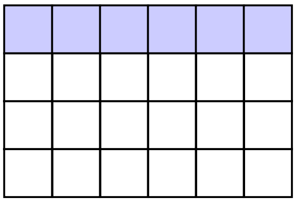


Any whole number can be used as a number base. ***Base 2***, which is also called ***binary***, uses just 0 and 1. ***Base*** *7* uses 0, 1, 2, 3, 4, 5, and 6. ***Base 16***, also called ***hexadecimal***, uses 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, and F. The hexadecimal number shown here is equal to 4,009 in base 10.



When there might be confusion as to which number base is being used, write the base as a subscript after the number, as in 305_7 or 469_{10}.

Fractions - Basics



$$= \frac{6}{24}$$

Numerator

Denominator

Every fraction has an endless list of ***equivalent fractions***, as shown below.

$$\frac{6}{24} = \frac{5}{20} = \frac{4}{16} = \frac{3}{12} = \frac{2}{8} = \frac{1}{4}$$

This fraction is in ***lowest terms*** because the only factor common to the numerator and denominator is 1.

Fractions can be written in several ways, and in each case the fraction bar means "divided by." Each fraction below equals 6 divided by 24, or 0.25.

$$\frac{6}{24} = 6/24 = {}^{6}/_{24} = 6 \div 24$$

This is an ***improper fraction***, because the numerator is larger than the denominator

$$\frac{59}{24} = 2\frac{11}{24}$$

This is the same number written as a ***mixed number***.

$$\frac{10}{27} \quad \frac{\overset{81}{3}}{\underset{80}{8}}$$

To determine whether two fractions are equal, cross-multiply as shown. If the two products are equal, the fractions are equal. If not, the fraction whose numerator is a factor in the larger product is the larger product. In this example, 81 > 80, so 3/8 > 10/27.

Fractions - Arithmetic

To add or subtract fractions, use a ***common denominator***.

$$\frac{2}{3} + \frac{1}{4} = \frac{8}{12} + \frac{3}{12} = \frac{11}{12}$$

$$\frac{2}{3} - \frac{1}{4} = \frac{8}{12} - \frac{3}{12} = \frac{5}{12}$$

To multiply fractions, multiply straight across:

$$\frac{2}{7} \text{ x } \frac{3}{5} = \frac{6}{35}$$

To divide fractions, multiply flip the second one and multiply:

$$\frac{2}{7} \div \frac{3}{5} = \frac{2}{7} \text{ x } \frac{5}{3} = \frac{10}{21}$$

When you flip a fraction, you get the its ***reciprocal***. If you multiply a fraction and its reciprocal, the product is always 1.

$$\frac{3}{4} \text{ x } \frac{4}{3} = \frac{12}{12} = 1$$

Percentages

A ***percentage*** is a fraction with the denominator 100:

$$31\% = \frac{31}{100} = 31 \text{ percent}$$

To convert a fraction into a percentage, multiply the numerator by 100, and divide by the denominator:

$$\frac{3}{5} = \frac{3 \text{ x } 100}{5}\% = 60\%$$

To convert a decimal to a percentage, move the decimal point two places to the right.

To convert a percentage to a decimal, move the decimal point two places to the left.

$$0.923 = 92.3\% \qquad 27.1\% = 0.271$$

To find a percentage increase or decrease, divide the change in value by the original value. For example, a $20 item on sale for $17 has changed by 3/20, or 15%.

Do not confuse a percentage with a ***percentage point***. For example, 5% is 150% more than 2%, even though it is only three percentage points more.

It is often helpful to use the chart on the right when setting up problems involving percentages. For example:

What is 30 percent of 250?

↓ ↓ ↓ ↓ ↓ ↓

X = 30 / 100 x 250

Word	**Meaning**
what	X (unknown)
is	=
percent	/100
of	x (times)

Money

American money uses ***dollars*** (***$***) and ***cents*** (***¢***), where 100¢ = $1.00.

Coin	Value (¢)	Value ($)
Penny	**1¢**	**$0.01**
Nickel	**5¢**	**$0.05**
Dime	**10¢**	**$0.10**
Quarter	**25¢**	**$0.25**
Half dollar	**50¢**	**$0.50**
Silver dollar	**100¢**	**$1.00**

If a person lends money, called ***principal***, for a period of time, the lender receives ***interest*** at an agreed upon ***interest rate*** from the borrower. The formula for calculating the amount of ***simple interest*** owed is:

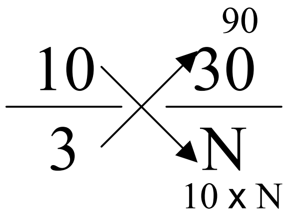
$$\text{Interest} = \text{Principal} \times \text{Interest Rate} \times \text{Time}$$

It is important to remember is that the unit of time must match the unit of the interest rate. For example, if you measure time in months, you must divide an annual interest rate by 12 in order to calculate interest appropriately.

Ratios and Proportions

A ***ratio*** describes the relationship between two quantities. For example, the ratio of legs to tails on a dog is 4 to 1, also written as 4:1 or 4/1.

A ***proportion*** is a statement that two ratios are equal. For example, 5:2 = 15:6 is a proportion. To solve a proportion with an unknown value, cross multiply:



$$10 \text{ x } N = 90 \longrightarrow N = 9$$

A ***scale*** is a ratio that changes the size of a drawing. For example, a drawing may be made on a scale of 100:1 in order to fit a large area onto one page.

Exponents

Base

Exponent

$$3^5 = 3 \times 3 \times 3 \times 3 \times 3 = 243$$

Three to the fifth ***power***.

If the exponent is 2, we say the number is ***squared***. For example, five squared is 25.

If the exponent is 3, we say the number is ***cubed***. For example, four cubed is 64.

$$3^2 \times 3^4 = 3^6$$

Multiply by adding exponents.

$$5^7 \div 5^4 = 5^3$$

Divide by subtracting exponents.

$$5^{-3} = 1/5^3$$

Negative exponents are reciprocals of positive exponents.

Zero is a special case. Any non-zero number to the zero power is 1, zero to any non-zero power is 0, but zero to the zero is undefined: $x^0 = 1$, $0^x = 0$, $0^0 =$ undefined.

As you multiply a number by itself, the ones digit follows a predictable pattern. For example, powers of 21 (21, 441, 9261, etc.) all end in 1. This is true for any number ending in 1 (31, 961, 29,791, etc.). Powers of 7 (7, 49, 343, 2401, 16,807, etc.) end in the repeating pattern 7, 9, 3, 1, 7, 9, and so on. The complete table is below.

Ones Digit	Pattern
0	0, 0, 0, 0, 0 . . .
1	1, 1, 1, 1, 1 . . .
2	2, 4, 8, 6, 2 . . .
3	3, 9, 7, 1, 3 . . .
4	4, 6, 4, 6, 4 . . .
5	5, 5, 5, 5, 5 . . .
6	6, 6, 6, 6, 6. . .
7	7, 9, 3, 1, 7 . . .
8	8, 4, 2, 6, 8 . . .
9	9, 1, 9, 1, 9 . . .

Roots

If A x A = B, then the ***square root*** of B is A. The square root of B is written as $\sqrt{B}$.
Every positive number B has both positive and negative square roots; for example, A x A = B and -A x -A = B.

If A x A x A = B, then the ***cube root*** of B is A. The cube root of B is written as $\sqrt[3]{B}$.

The nth root of a number x is a number, that when raised to the nth power, gives x:

$$\sqrt[n]{x} = y \quad \longleftrightarrow \quad y^n = x$$

$$5^{1/2} = \sqrt{5}$$

Fractional powers are roots.

$$\sqrt{ab} = \sqrt{a} \times \sqrt{b}$$

The square root of a product is the product of the roots.

$$\sqrt{72} = \sqrt{36} \times \sqrt{2}$$
$$\sqrt{72} = 6\sqrt{2}$$

This example illustrates how square roots that include perfect squares are simplified.

$$\frac{4}{\sqrt{7}} \times \frac{\sqrt{7}}{\sqrt{7}} = \frac{4\sqrt{7}}{7}$$

As a general rule, do not write fractions with roots in the denominator. Multiply the numerator and the denominator to change the form of the fraction.

Order of Operations

If you have have an expression with several operations, the evaluation goes in the following order:

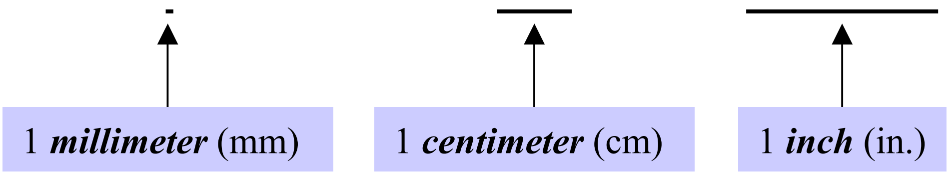
1) Parentheses
2) Exponents
3) Multiplication and Division
4) Addition and Subtraction

Multiplication and division are on the same level; they proceed from left to right. Similarly, addition and subtraction go from left to right.

Example: $3 + (9 – 4) - 2 \times 5 + 1 \times 2^3$

Order	Rule	Result
1	Evaluate inside the parentheses.	$3 + \mathbf{5} - 2 \times 5 + 1 \times 2^3$
2	Simplify the exponents.	$3 + 5 - 2 \times 5 + 1 \times \mathbf{8}$
3	Do the leftmost multiplication or division.	$3 + 5 - \mathbf{10} + 1 \times 8$
4	Continue doing multiplication and division.	$3 + 5 - 10 + \mathbf{8}$
5	Do the leftmost addition or subtraction.	$\mathbf{8} – 10 + 8$
6	Continue doing addition and subtraction.	$\mathbf{-2} + 8$
7	Continue doing addition and subtraction.	$\mathbf{6}$

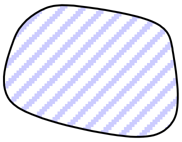
Units of Distance



U.S. Customary Units		
1 ***foot*** (ft.)	=	12 in.
1 ***yard*** (yd.)	=	3 ft.
1 ***mile*** (mi.)	=	5,280 ft.
Metric Units		
1 cm	=	10 mm
1 ***meter*** (m)	=	100 cm
1 ***kilometer*** (km)	=	1,000 m
Conversions		
1 in.	=	2.54 cm
1 m	=	39.37 in.
1 mi.	=	1.609 km

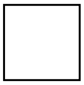
Units of Area and Volume

Just as units of length describe how long something is, units of ***area*** describe how much surface a shape covers. Many units of area are squares of unit length:

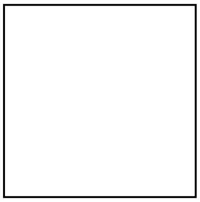


area

1 ***square millimeter*** (mm^2)



1 ***square centimeter*** (cm^2)

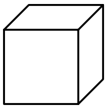


1 ***square inch*** (in.^{2})

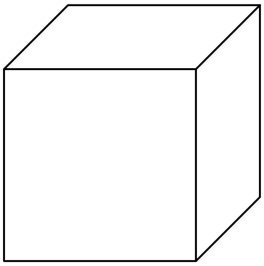
144 in.^{2} = 1 ***square foot*** (ft.^{2})
9 ft.^{2} = 1 ***square yard***
43,560 ft.^{2} = 1 ***acre***

Units of ***volume*** describe how much space an object occupies. Many units of area are cubes of unit length:

1 ***cubic millimeter*** (mm^3)



1 ***cubic centimeter*** (cm^3)



1 ***cubic inch*** (in.^{3})

A cubic centimeter is equivalent to a ***milliliter*** (ml), as 1,000 ml = 1 ***liter*** (l).

In addition to cubic inches, the U.S. Customary System uses ***gallons*** (g) to measure volume.

1 gallon = 4 ***quarts*** (qt.)
1 quart = 2 ***pints*** (pt.)
1 pint = 2 ***cups*** (C.)

A gallon is also approximately equal to 3.78 liters.

Prefixes

The basic units of measure, especially in the metric system, can be modified by the use of prefixes. For example, a kilometer equals 1,000 meters, and a milligram equals 0.001 grams.

Prefix	As Exponent	As Number
Pico	10^{-12}	0.000000000001
Nano	10^{-9}	0.000000001
Micro	10^{-6}	0.000001
Milli	10^{-3}	0.001
Centi	10^{-2}	0.01
Deci	10^{-1}	0.1
Deca	10^{1}	10
Hecto	10^{2}	100
Kilo	10^{3}	1,000
Mega	10^{6}	1,000,000
Giga	10^{9}	1,000,000,000
Tera	10^{12}	1,000,000,000,000

Temperature

Temperature is measured on two scales, ***Fahrenheit*** (F)and ***Celsius*** (C), also known as ***centigrade***. Both scales use ***degrees*** as the unit of measure, but a Celsius degree is larger than a Fahrenheit degree.

	Fahrenheit	**Celsius**
Water Freezes	32° F	0° C
Water Boils	212° F	100° C

To convert between Fahrenheit temperatures (F) and Celsius temperatures (C), use the following formulas:

$$\mathbf{F = {}^{9}/_{5}\, C + 32}$$

$$\mathbf{C = {}^{5}/_{9}\, (F - 32)}$$

Mass and Weight

U.S. Customary Units		
1 ***pound*** (lb.)	=	16 ***ounces*** (oz.)
1 ***ton***	=	2,000 lbs.
Metric Units		
1 ***gram*** (g)	=	1,000 ***milligrams*** (mg)
1 ***kilogram*** (kg)	=	1,000 grams
Conversions		
1 oz.	=	28.35 g
1 lb.	=	453.6 g
1 kg	=	2.2 lbs.

Time

The basic unit of time is the ***second*** (sec.). Other units of time are based on the second.

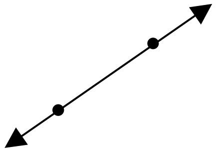
1 ***minute*** (min.)	=	60 seconds
1 ***hour*** (hr.)	=	60 minutes
1 ***day***	=	24 hours
1 ***week***	=	7 days
1 ***month***	=	28 – 31 days
1 ***year***	=	12 months, or 365 days (approx.)

The ***rate*** (r) at which an object moves equals the distance (d) that it moves divided by the length of time (t) that it moves. That is, **r = d/t**. Similarly, the distance that it moves equals the rate at which it moves times the length of time; **d = r x t.**

Geometry – The Basics

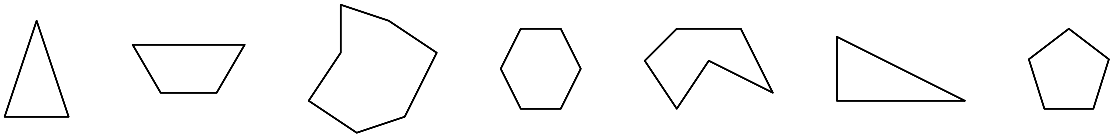
A ***point*** is a location. We represent it with a small dot (as shown to the right), but a point actually has no size at all.

There is exactly one straight line that goes through any two points.



Three points define a ***plane***, which is like a flat surface that extends forever. You can think of a plane as being a table top that never ends, but a plane has no thickness.

A ***polygon*** is a closed figure with straight sides. Below are several examples.



Polygons are named according to how many sides they have, and the sum of the angles in a polygon of n sides is 180(n-2) degrees, as shown in the tables below.

Sides	Name	Angle Sum
3	Triangle	180°
4	Quadrilateral	360°
5	Pentagon	540°
6	Hexagon	720°

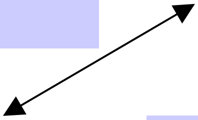
Sides	Name	Angle Sum
7	Heptagon	900°
8	Octagon	1080°
9	Nonagon	1260°
10	Decagon	1440°

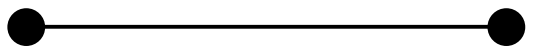
The sum of the exterior angles of a polygon (the supplementary angles of the interior angles) is always 360°.

Two polygons, line segments or angles are ***congruent*** if they are the exact same size and shape. A polygon is ***regular*** if all of its sides are the same length.

Lines

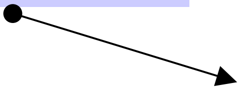
A ***line*** extends forever in both directions.

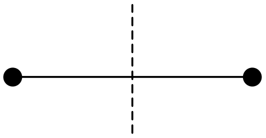




A ***line segment*** is of limited length.

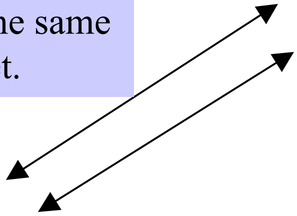
A ***ray*** extends forever in the direction away from its ***endpoint***.

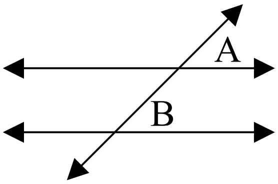




A ***segment bisector*** cuts a line segment in half.

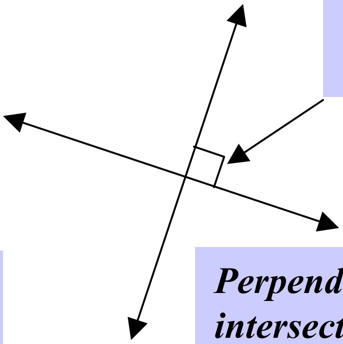
Parallel lines point in the same direction and never meet.





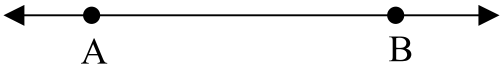
Corresponding angles formed by a line intersecting parallel lines (such as A and B) have the same measure. The line that crosses the others is a ***transversal.***

This symbol indicates a 90° (***right***) angle.

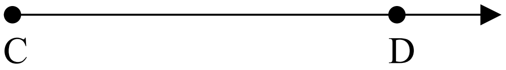


Perpendicular lines meet, or ***intersect*** at a 90° angle.

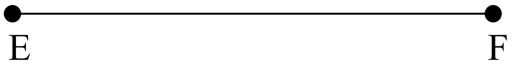
$\overleftrightarrow{AB}$ Line AB



$\overrightarrow{CD}$ Ray CD



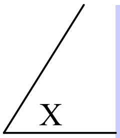
$\overline{EF}$ Line Segment EF



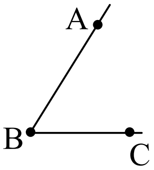
$\overleftrightarrow{WX} \parallel \overleftrightarrow{YZ}$ means that line WX ***is parallel to*** line YZ.

$\overleftrightarrow{WX} \perp \overleftrightarrow{YZ}$ means that line WX ***is perpendicular to*** line YZ.

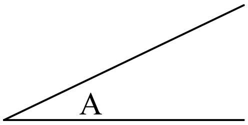
Angles



We can name an angle by a letter: “∠X” means “angle X.”

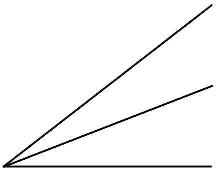


We can name an angle by the points that form it: “∠ABC” means “the angle formed by going from A to B to C.” The “corner” of the angle, in this case B, is called the ***vertex*** of the angle.

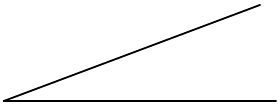


If angle A measures 29 degrees, we write $m\measuredangle A = 29°$, which is read as “the measure of angle A is 29 degrees.”

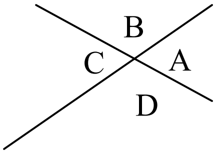
A ***right angle*** measures exactly 90°.



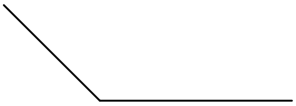
Angles that share a side are called ***adjacent angles***.



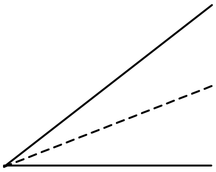
An ***acute angle*** is smaller than 90°.



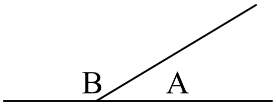
Angles A and C are a pair of ***vertical angles***. So are angles B and D.



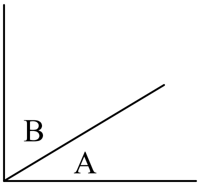
An ***obtuse angle*** is more than 90°.



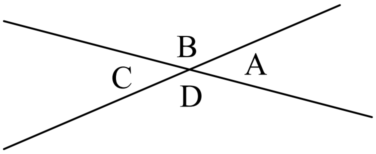
An ***angle bisector*** cuts an angle in half.



Supplementary angles add up to 180°



Complementary angles add up to 90°

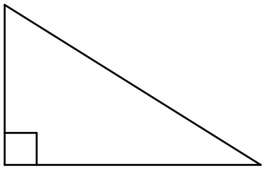


$m\angle A = m\angle C \quad \text{and} \quad m\angle B = m\angle D$

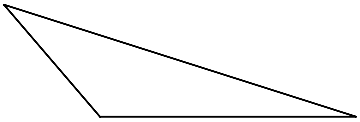
$m\angle A + m\angle B = m\angle C + m\angle D = 180°$

$m\angle A + m\angle B + m\angle C + m\angle D = 360°$

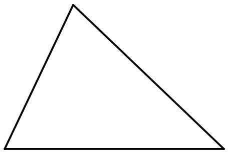
Triangles



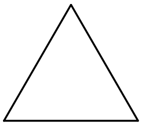
A ***right triangle*** has a right (90°) angle.



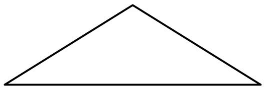
An ***obtuse triangle*** has an angle whose measure is larger than 90°.



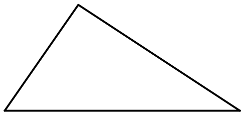
An ***acute triangle*** has three angles smaller than 90°.



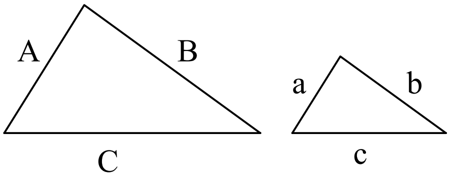
An ***equilateral triangle*** has three sides that are the same length, and three 60° angles.



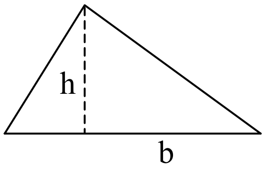
An ***isosceles triangle*** has two sides that are the same length, and two angles with the same measure..



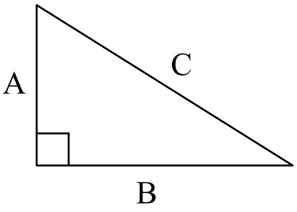
A ***scalene triangle*** has no two sides with the same length.



Two ***similar triangles*** have the same angle measures, and A/a = B/b = C/c.



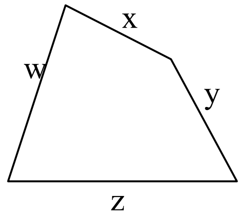
The ***area*** (A) of a triangle is the amount of surface it covers, calculated as $A = \frac{1}{2} bh$. The ***perimeter*** (P) of a triangle (or any other polygon) is the sum of the lengths of its sides.



Pythagorean Theorem: In a right triangle, if A and B are the lengths of the two short sides, and C is the length of the ***hypotenuse*** (long side), then $A^2 + B^2 = C^2$.

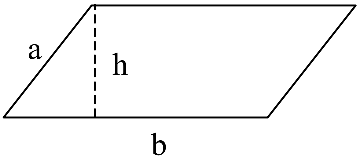
Quadrilaterals

Like triangles, quadrilaterals have perimeter (P) and area (A).



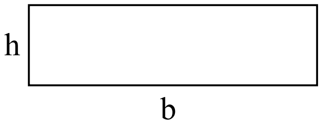
A ***quadrilateral*** is a closed polygon with four sides.

$$P = w + x + y + z$$



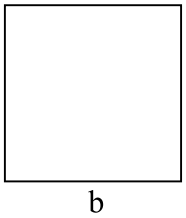
A ***parallelogram*** has two pairs of parallel sides. In these formulas, b is the length of the ***base***, and h is the ***height***.

$$A = b \times h \qquad P = 2(a+b)$$



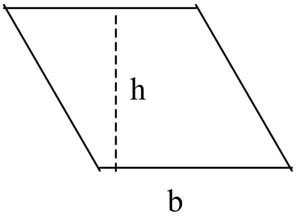
A ***rectangle*** is a parallelogram with four 90° angles.

$$A = b \times h \qquad P = 2(b+h)$$



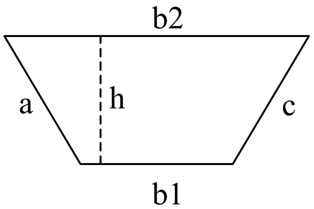
A ***square*** is a rectangle whose sides are all the same length.

$$A = b^2 \qquad P = 4b$$



A ***rhombus*** is an equilateral parallelogram.

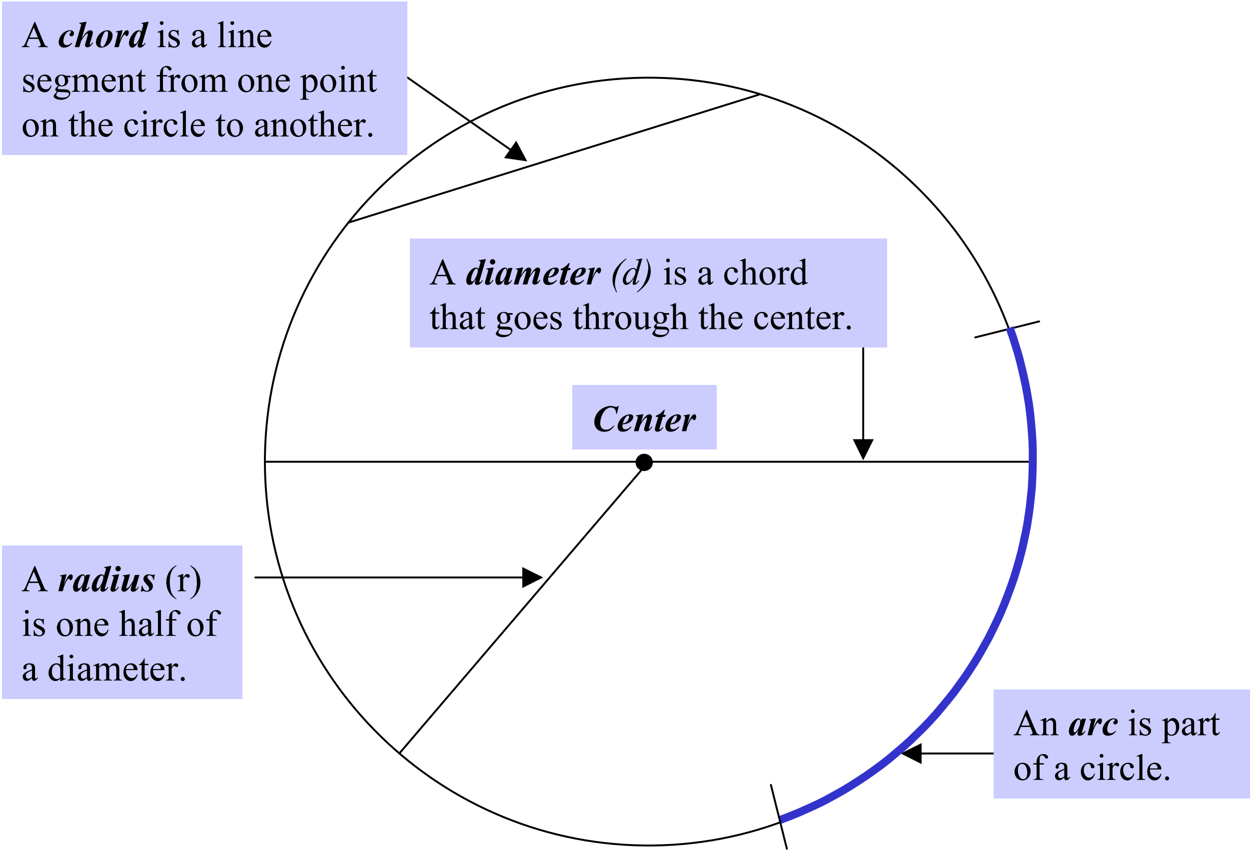
$$A = b \times h \qquad P = 4b$$



A ***trapezoid*** is a quadrilateral with one pair of parallel sides.

$$A = \tfrac{1}{2}\,(b1+b2) \times h \qquad P = b1 + b2 + a + c$$

Circles



A ***chord*** is a line segment from one point on the circle to another.

A ***diameter*** *(d)* is a chord that goes through the center.

A ***radius*** (r) is one half of a diameter.

An ***arc*** is part of a circle.

A ***circle*** is a closed, two-dimensional shape where every point is the same distance from another point, called the center.

A ***semicircle*** is half of a circle.

The ***circumference*** (C) of a circle is the distance around the circle.

The number ***pi*** (π) is the circumference divided by the diameter: $\pi = C / d$

$\pi = 3.1415926\ldots$ and **π is approximately** $^{22}/_{7}$

The diameter of a circle is twice as long as the radius: $d = 2 \times r$

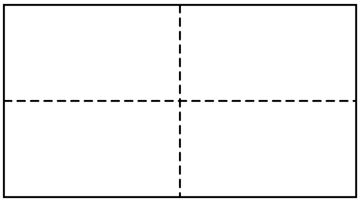
The area (A) of a circle is calculated as follows: $A = \pi \times r^2$

The formulas for circumference are: $C = \pi \times d$ and $C = 2 \times \pi \times r$

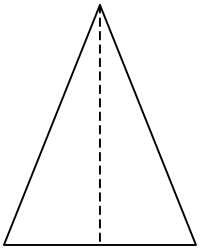
There are 360 degrees in a circle.

Symmetry

An object has ***mirror symmetry*** if you can split it with a line segment, called a ***line of symmetry***, into two halves that are mirror images of one another. Mirror symmetry is also called ***reflection symmetry*** or ***line symmetry***.

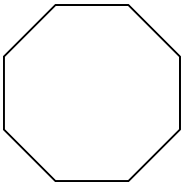


A rectangle has two lines of symmetry.

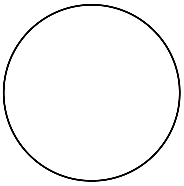


An isosceles triangle has one lines of symmetry.

An object has ***rotational symmetry*** if you can rotate it less than 360° and produce a shape identical in size and orientation to the original.



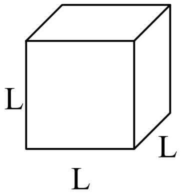
An equilateral octagon, rotated by any multiple of 45 degrees, is identical to the original position.



A circle, rotated by any amount, looks the same..

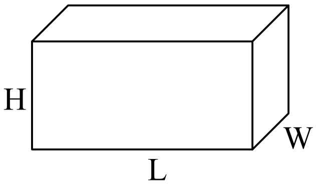
3D Shapes

Three dimensional shapes have the properties of ***surface area*** (SA) and ***volume*** (V). Surface area is the area the outer surface would cover if it were laid flat. Volume is the amount of space the object occupies.



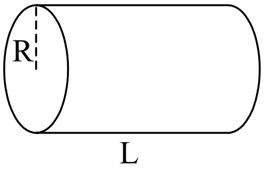
A ***cube*** is the same length on all sides; all angles are 90°.

SA = 6 X L^2 **V = L^3**



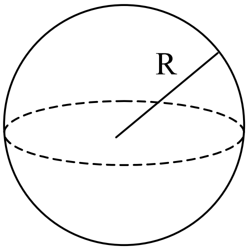
A ***rectangular solid*** is similar to a cube, but the length, width, and height are different.

SA = 2 (LW+LH+WH) **V = LWH**



A ***cylinder*** is shaped like a can, with a circle at both ends. It may be helpful to think of a cylinder as a stack of circles.

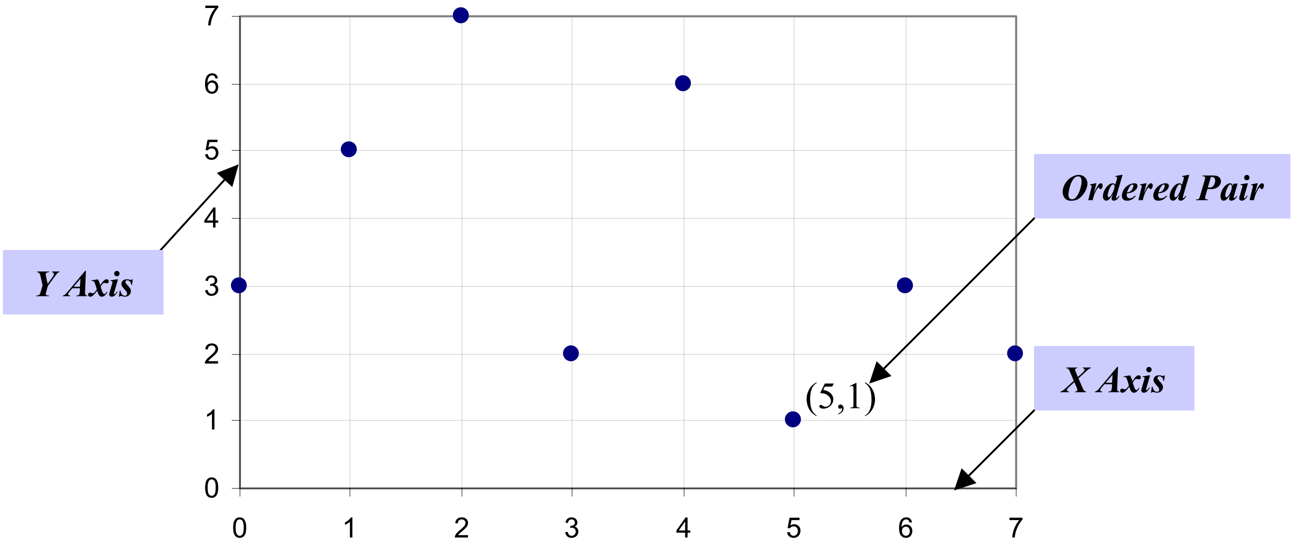
SA = $2\pi R(L+R)$ **V = $\pi R^2 L$**

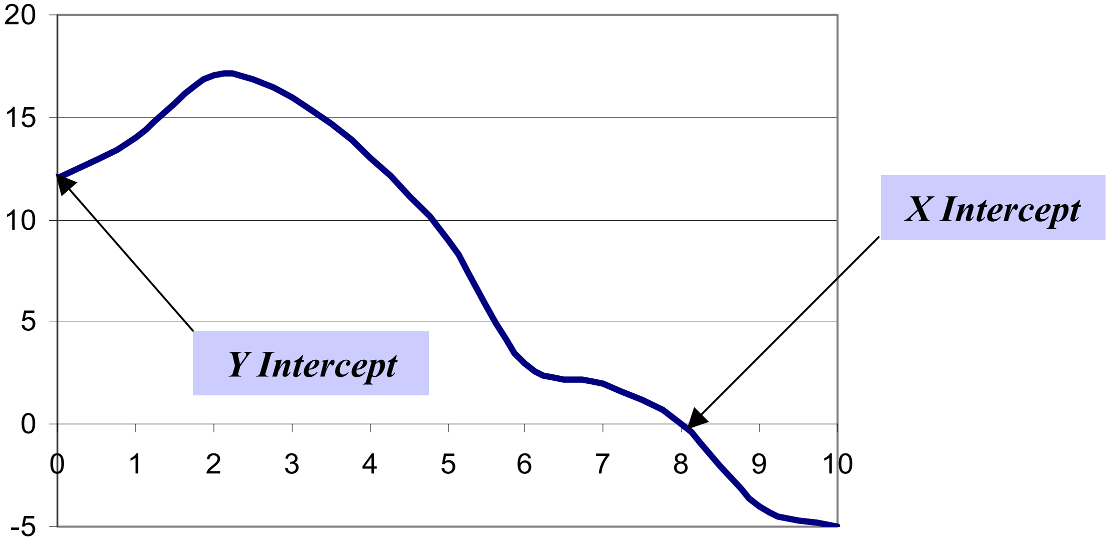


A ***sphere*** is a ball-shaped object. Every point on the outer surface of the sphere is the same distance, R, from the center.

SA = $4\pi R^2$ **V = $^4/_3\pi R^3$**

X-Y Coordinates

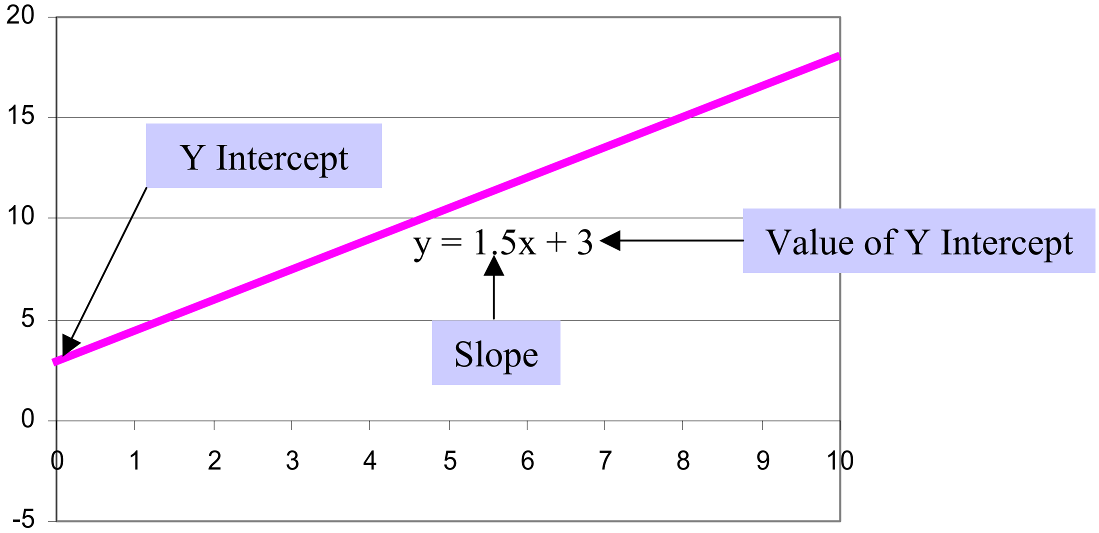




An X intercept is a point at which the Y value of a graph is 0, and it is always of the form (x,0).

A Y intercept is a point at which the X value of a graph is 0, and it is always of the form (0,y).

Equations of Lines



The ***slope*** of a line is the amount the Y value changes when the X value increases by 1.

The formula for the slope of a line containing points (x_1, y_1) and (x_2, y_2) is:

$$\textbf{slope} = (y_2 - y_1)/(x_2 - x_1)$$

For example, the line shown above contains the points (0,3) and (8,15), so its slope is (15-3) / (8-0), or 1.5.

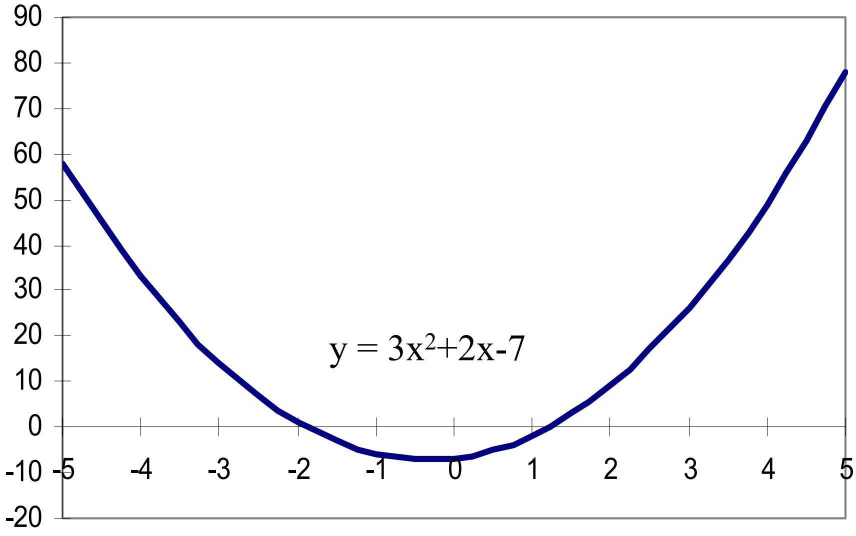
Lines with positive slopes move up as they go to the right; lines with negative slopes move up as they go to the left.

The ***slope-intercept form*** of the equation of a line is **y = mx+b**, where m is the slope of the line, and b is the value of the y intercept.

Two lines are parallel if they have the same slope.

Two lines are perpendicular if their slopes are negative reciprocals of each other. For example, lines with slopes of $^5/_3$ and $-^3/_5$ are perpendicular to one another.

Quadratic Polynomials



A ***quadratic*** polynomial is one whose highest power is 2. For example, $3x^2+2x-7$ is a quadratic polynomial.

You can sometimes find the ***root***, or ***zero***, of a quadratic polynomial by factoring. For example, you can factor $x^2 – 4x – 21$ into $(x-7)\ (x+3)$. From this, we see that the quadratic is 0 if and only if $x=7$ or $x=-3$.

You can also find roots of the quadratic polynomial $ax^2 + bx + c$ by using the ***quadratic equation***:

$$\mathbf{x = \frac{-b\ ^{+}/-\ \sqrt{b^2 - 4ac}}{2a}}$$

Probability

The ***probability*** of an event is a number from 0 to 1 that describes the likelihood of an event. A probability of 0 means that an event cannot happen, and a probability of 1 means the event is certain to happen.

If the probability of an event is x, the probability of the event not happening is 1 – x.

If there are several equally likely outcomes for an event, the probability of any given outcome is 1 divided by the number of outcomes:

Probability of getting heads on a coin flip = ½
Probability of rolling a 4 on a six-sided die = $^1/_6$
Probability of drawing a club from a deck of cards = ¼

If the probability of one event is not affected by another event, the two events are ***independent.*** For example, the probability of drawing the king of hearts from a deck of cards is not affected by whether a coin flip turns up heads or tails.

If the probability of one event is affected by the another event, the two events are ***dependent***. For example, the probability of a man weighing over 200 lbs. is dependent on whether the person is over six feet tall, because tall people are generally heavier than short people.

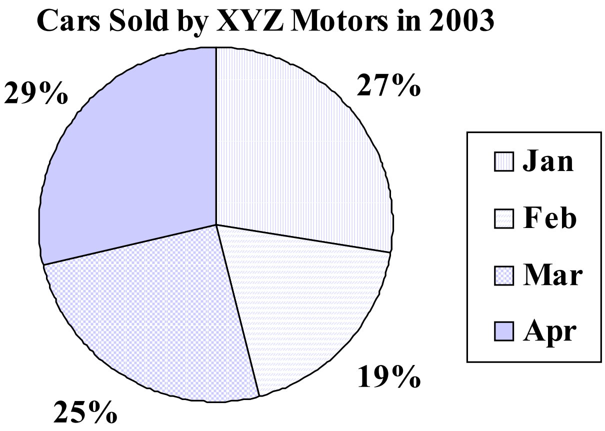
If two events are independent, and the probabilities of each one happening are x and y, the probability of both events is xy. For example, the probability of drawing a heart from a deck of cards and getting a coin flip to land "heads" is $^1/_4 \times {}^1/_2$, or $^1/_8$.

Charts and Tables

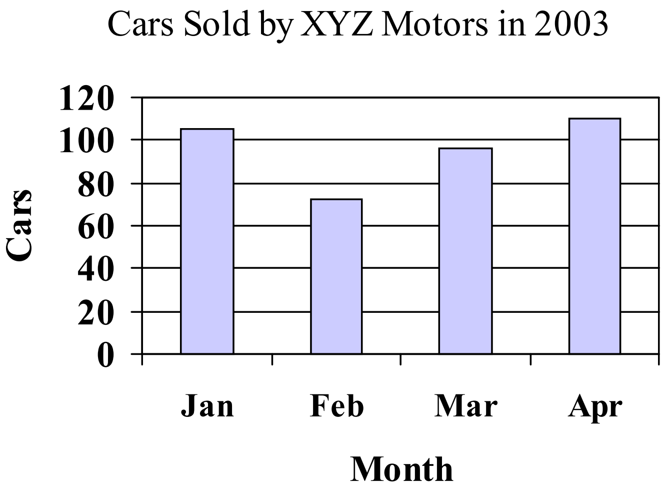
Cars Sold by XYZ Motors in 2003

Month	**Cars**
Jan	105
Feb	72
Mar	96
Apr	110

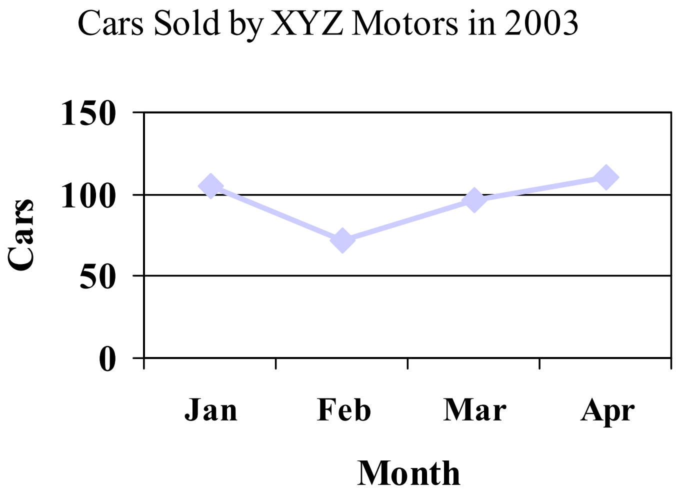
Table



Pie (Circle) Chart



Bar Chart



Line Chart

The table and all of the charts have titles describing what the numbers represent. Also, note that the bar and line charts have both labels and titles on the horizontal and vertical axes. The horizontal lines on these two charts are optional, and they assist the reader in figuring out what the values are. Finally, note that the pie chart has a legend indicating what month each slice represents.

Statistics

There are three ways to summarize the central tendencies of a list of numbers:

To find a ***mean,*** add up the numbers and divide by the number of numbers.
The ***median*** is the middle number when the list is written in numerical order.
The ***mode*** is the number that appears most frequently in the list.

The mean, median, and mode are all ***averages***, but when somebody refers to the average of a group of numbers, the one meant is usually the mean.

If the number of items in a list is even, the median is halfway between the two middle numbers. For example, the median of 3, 5, 6, 7, 12, and 15 is 6.5.

2, 3, 3, 3, 3, 5, 6, 7, 9, 10, 10, 10, 20

For the list of numbers shown above:
Mean = (2+3+3+3+3+5+6+7+9+10+10+10+20)/13 = 7
Median = 6 (middle item in the numerically ordered list)
Mode = 3 (number that appears most frequently on the list)

The word ***range*** has two meanings. In this list: 3, 6, 11, 10, 5 we could say the range is from 3 to 11, or we could say the range is 8, because 11-3 = 8.

The word ***quartile*** has two meanings, both of which are based on dividing a numerically ordered list of numbers into four parts with equal quantities of numbers in each part:

| 1, 2, 2, 3, 3, | 4, 5, 6, 7, 11, | 13, 14, 14, 15, 15, | 18, 20, 20, 21, 23 |

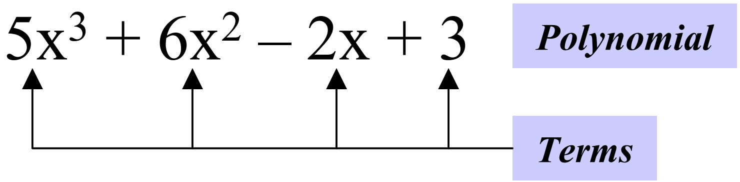
3.5 12 16.5

In the first meaning, the four quartiles are {1, 2, 2, 3, 3}, {4, 5, 6, 7, 11}, {13, 14, 14, 15, 15}, and {18, 20, 20, 21, 23}.

In the second meaning, the three quartiles are 3.5, 12, and 16.5, because these numbers are halfway between the ends of the various quartiles.

Percentiles are like quartiles, but the ordered number list is split into 100 percentiles, rather than 4 quartiles. Similarly, ***quintiles*** are based on the list being split into 5 parts.

Polynomials and Functions



$$5x^3 + 6x^2 - 2x + 3$$

Coefficients are the numbers in front of the variables. In the polynomial above, the coefficients are 5, 6, and –2.

You can simplify a polynomial by adding terms that have the same variable powers. For example:

$$3x^4 + 2x^2 - 5y^2 - 7x + 2 + 6x^4 + 3x^2 + 5x - 2 = 9x^4 + 5x^2 - 5y^2 - 2x$$

To multiply a polynomial by a number, multiply each term in the polynomial by the number:

$$5\ \mathrm{x}\ (x^2 - 4xy + 3y^2 - 2) = 5x^2 - 20xy + 15y^2 - 10$$

$f(x) = 3x^2 - 4x + 8$ ***Function***

x	f(x)
-3	47
-2	28
-1	15
0	8
1	7
2	12
3	23

Sets

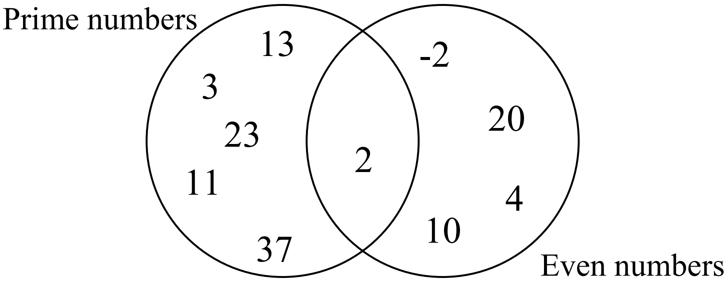
A ***set*** is a collection of things. Each thing in the set is called an ***element*** of the set. Sets can be described in three ways:

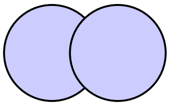
{red, green, blue}	An ***enumeration***, or list
{people named Rita}	An informal description of a rule
$\{x \mid 3 < x < 8\}$	A mathematical description of a rule, in this case numbers greater than 3 but less than 8

The symbols $\in$ and $\notin$ mean “is an element of” and “is not an element of,” respectively.

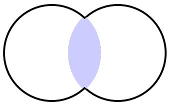
For example, $3 \in$ {odd numbers}, and $4 \notin$ {odd numbers}.

A ***Venn diagram*** shows how two sets are related:





Union

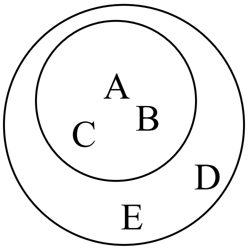


Intersection

The union of sets X and Y is written as $X \cup Y$.

The intersection of sets X and Y is written as $X \cap Y$.

If every element in a set is also in a second set, the first set is a ***subset*** of the second.



$\{A, B, C\} \subset \{A, B, C, D, E\}$ means that the first set is a subset of the second.

A set with no elements is called an ***empty set***, with the symbol $\varnothing$. As an example, $\varnothing =$ {Ants weighing 1,000 lbs.}

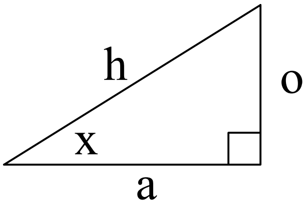
Properties of Real Numbers

Name	Formal Statement	Meaning
Reflexive	$a = a$	Any number equals itself.
Commutative	$a + b = b + a$ $a \times b = b \times a$	The order in which you add or multiply two numbers does not matter.
Associative	$a + (b + c) = (a + b) + c$ $a \times (b \times c) = (a \times b) \times c$	Moving parentheses around does not change the results of addition or multiplication.
Distributive	$a(b + c) = ab + ac$	Multiplying one number by the sum of two numbers is equivalent to multiplying the first number by each of the other two and then adding.
Transitive (Inequality)	If $a<b$ and $b<c$, then $a<c$ If $a>b$ and $b>c$, then $a>c$	If one number is less than a second, and the second number is less than a third, then the first number is less than the third. A similar rule holds for “greater than.”
Transitive (Equality)	If $a=b$ and $b=c$, then $a=c$	If two numbers are both equal to a third number, then they are also equal to each other.
Additive Identity	$a + 0 = a$	Adding zero to any number leaves the number unchanged.
Multiplicative Identity	$a \times 1 = a$	Multiplying any number by one leaves the number unchanged.
Additive Inverse	$a + -a = 0$	Any number, added to its negative, equals zero.
Multiplicative Inverse	$a \times 1/a = 1$	Any number (other than zero), multiplied by its reciprocal, equals one.

Trigonometry

Let x be an angle, and let:

o = opposite side's length
a = adjacent side's length
h = hypotenuse's length



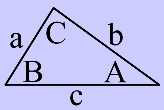
Name	Form	Def'n.	0°	30°	45°	60°	90°
sine	sin(x)	o/h	0	1/2	sr(2)/2	sr(3)/2	1
cosine	cos(x)	a/h	1	sr(3)/2	sr(2)/2	1/2	0
tangent	tan(x)	o/a	0	sr(3)/3	1	sr(3)	Und
secant	sec(x)	1/cos(x)	1	2sr(3)/3	sr(2)	2	Und
cosecant	csc(x)	1/sin(x)	Und	2	sr(2)	2sr(3)/3	1
cotangent	cot(x)	1/tan(x)	Und	sr(3)	1	sr(3)/3	0

Abbreviations: sr( ) → "square root" Und → "undefined"

"sohcahtoa" stands for sine = opp/hyp cosine = adj/hyp tangent = opp/adj

Notation: $\mathbf{(\sin(x))^n = \sin^n(x)}$. Similar notation applies to cos, tan, sec, csc, and cot.

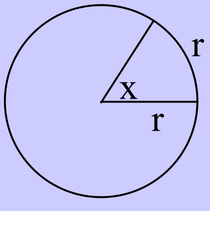
For any angle x, $\mathbf{\sin^2(x) + \cos^2(x) = 1}$



Law of Sines: sin(A)/a = sin(B)/b = sin(C)/c

Law of Cosines: $c^2 = a^2 + b^2 - 2ab \times \cos(C)$

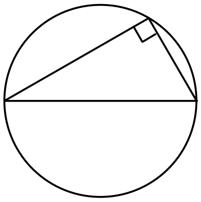
A **radian** is a measure of an angle, such that if the angle were a central angle of a circle, the length of the arc it intercepted would be equal to the circle's radius.



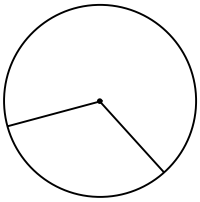
Angle x measures one radian

2π radians = 360°

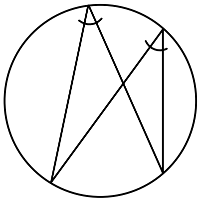
Circle Theorems



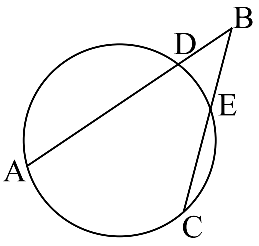
Any angle that subtends a diameter and has its vertex on the circle measures 90°.



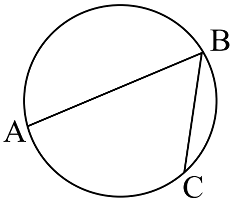
Any arc measures the same as the central angle subtending it.



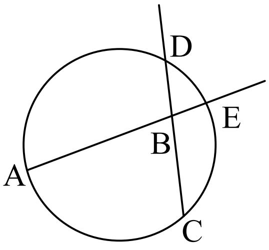
Angles that subtend the same arc and have vertices on the circle are equal.



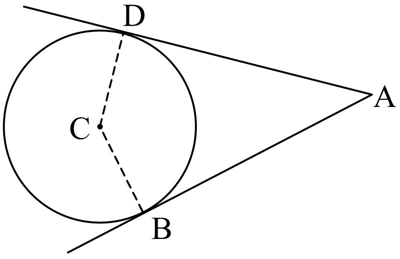
$m\angle ABC = (m\,\overset{\frown}{AC} - m\,\overset{\frown}{DE})/2$



$m\angle ABC = (m\,\overset{\frown}{AC})/2$



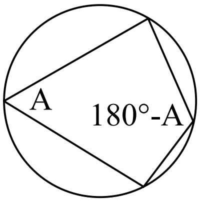
$m\angle ABC = (m\,\overset{\frown}{AC} + m\,\overset{\frown}{DE})/2$



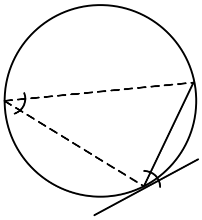
If two line segments are tangent at points B and D to the circle with center C, then:

AD = AB

$m\angle ABC = m\angle ADC = 90°$



Opposite angles of a quadrilateral inscribed in a circle total 180°.



The angle between any chord and one of its tangents equals any inscribed angle subtending the chord.